ARTICLES

The Incompleteness of Central Planning

Hai-Trieu v. Nguyen[1][a]

[1] Independent

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This article aims to flesh out the implication of Kurt Gödel's incompleteness theorems for the computability of social economic planning. Specifically, the article challenges the claim of modern technosocialists that access to large sets of economic data allows the social planner (or, equivalently, a supercomputing machine) to solve the planning problem for the economy. The article extends the computability problem proposed by the literature on computable economics for market socialism to the case of technosocialism. The conclusion is that even if all the practical challenges could be overcome, social economic calculation and planning are still impossible from a computation-theoretic point of view.

> A completely unfree society (i.e., one proceeding in everything by strict rules of "conformity") will, in its behavior, be either inconsistent or incomplete, i.e., unable to solve certain problems, perhaps of vital importance. Both, of course, may jeopardize its survival in a difficult situation.
>
> —Kurt Gödel (quoted in Wang 1997, 4)
>
> Mises was way ahead of his time in thinking about computational complexity.
>
> —Glen Weyl (2019)

The socialist calculation debate is usually considered a concluded chapter in the history of economic thought, even if there still exist different interpretations of the debate. On the one hand, proponents of Ludwig von Mises (2012) and Friedrich Hayek's (1945) thesis believe that critics never fully resolved the calculation problem proposed by Mises in 1920 and the

a Hai-Trieu v. Nguyen (nguyenviethaitrieu@gmail.com) is an independent scholar based in Saigon, Vietnam. He holds a master's degree in economics from the University of California, Santa Barbara.

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knowledge problem proposed by Hayek in 1945 (Boettke and Candela 2023; Lambert and Fegley 2023; Lavoie 1985). On the other hand, theorists in the opposite camp firmly believe that the proposals of Oskar Lange (1936, 1937) and Abba Lerner (1934) for market socialism did indeed overcome all the theoretical challenges and that the practical matters to be resolved are merely those of information transmission and the computational capabilities of the social planner (Hurwicz 1969, 1973). Inevitably, with the recent rapid development of data mining and advanced computing techniques such as statistical learning and nonlinear optimization, a new wave of advocates for collective economic planning based on complex informational and algorithmic control, or technosocialism, has emerged (Cockshott and Cottrell 1993; Dapprich and Cockshott 2023). The advocates of technosocialism essentially argue that the informational and technological deficits of the past are no longer a concern, since modern computational advances now allow the central planner to effectively calculate economic decisions on behalf of economic agents.

Proponents of the Mises-Hayek thesis on the infeasibility of socialism have offered a lengthy response to the proponents of technosocialism (see Boettke and Candela 2023; Lambert and Fegley 2023; Moreno-Casas, Espinosa, and Wang 2023). Their shared main thesis is that the challenge faced by the economic planner is never an informational or even a computational problem, but indeed an economic and institutional problem. They all seem to agree with Hayek's (1945) assertion that if we assume that somehow all relevant contextual and institutional knowledge at all times and places could be gathered and given, and that somehow the central planner has a device to convert all ordinal preferences to cardinal utility functions, then the planning problem could, at least theoretically, be solved by hypothesis.

Unfortunately, throughout both the socialist calculation debate and current discussions, attention has been directed away from the computational capabilities of the economic planner. It was initially admitted that computational insufficiency was a handicap for market socialism, but such a drawback seems to be overcome with the aid of advanced computing machines and techniques. When attention is given to the computability problem, the problem is seen more as practical challenge (whether information can be aggregated and processed to yield a solution, or whether the solution can be calculated within a realistic time window) than as a theoretical challenge (whether the central planner could come up with an optimal calculation *even if* all necessary information were given) (Engelhardt 2013; Cwik and Engelhardt 2023).

This article offers a theoretical challenge to the computability problem. The computability problem of market socialism has long been proposed by the literature on computable economics (Velupillai 2000; Wolpert 2001; Koppl and Rosser 2002). We maintain that modern technosocialism cannot escape from the same computability problem that its predecessor, market socialism, faced in the past. It must be noted that the theoretical computability problem discussed here is different from the practical computation problem highlighted in Engelhardt (2013) and Cwik and Engelhardt (2023). The argument of this article relies heavily on the incompleteness theorems of Kurt Gödel, which imply that no machine can replace the human mind when it comes to computation. The conclusion is that even under an ideal scenario such that all relevant economic information can be gathered as correct inputs for central planning, technosocialism is still impossible because it cannot compute a solution for the planning problem of the whole economy.

Kurt Gödel is widely considered the most influential mathematician and logician of the twentieth century. Among other contributions, his two incompleteness theorems, which proved the inexhaustibility of mathematics, are regarded as the most important results in mathematical logic to date. Despite their impacts in computer science, especially in complexity theory, their implication for the social sciences has not been widely studied. In fact, an extension of the theorems to the case of socialism is not far-fetched, since Gödel himself proposed a “social” version of his theorems, as quoted at the beginning of the article, although he never tried to present a formal proof for it. This article hopes to present the direct relevance of Gödel’s theorems to the case of technosocialism. Though Hayek (1963, 341) recognized the parallel between Gödel’s arguments and his own, this similarity has never been explicitly considered. The claims of this article are twofold: first, that Gödel’s incompleteness theorems, when applied to the computability problem faced by technosocialism, share many epistemological similarities with the Mises-Hayek thesis; and second, that the nature of economic computation is so complex that regardless of how the modern technosocialists try to adjust their models (e.g., Dapprich and Greenwood 2024), social economic planning is still computationally impossible.

The first section restates some foundational definitions and results in computation theory along with Gödel’s incompleteness theorems and their implication within a computational context. The second section presents literature on computable economics that shows how the computability problem effectively refutes market socialism. The third section summarizes the current state of the technosocialist calculation debate, and the fourth section extends the computability problem to the case of technosocialism. Because this article’s central focus is on the computability problem, it tries to avoid economic and institutional arguments; thus, they are presented only for the sake of comparison. This serves to highlight the article’s fundamental—and more extreme—argument that even if all relevant economic information could be gathered as correct inputs, and even given computational sufficiency, it is still impossible for the planner to effectively compute a solution to the planning problem.

Computability and Unsolvability

In this section we restate some foundational concepts in computation theory, a detailed treatment of which could be found in any textbook on mathematical logic, such as Hedman (2006) or Leary and Kristiansen (2015).

The Church-Turing Thesis, Mathematical Formulation

A mathematical problem is defined to be *computable* or *decidable* if there exists an algorithm that is capable of solving the problem. Likewise, a function is *computable* if there exists an algorithm that can carry out the task of the function: receiving an input and returning an output. Solvability and computability hence depend on the existence of an algorithm, which conventionally is understood as the operation of a *Turing machine*. The Turing machine is an idealized computing device described by Alan Turing (1937) such that given an infinite tape (storage space), a finite number of instructions, and an input, it records the output on the tape at every step. A problem is solvable if the Turing machine stops, or halts, after a finite number of steps and returns an output on the tape. A computable problem is thus a problem that could be computed by the Turing machine. Consequently, computability refers to *Turing computability*. Since computability is defined as the halting, or termination, of the Turing machine, a problem is called *unsolvable* or *noncomputable* if the Turing machine does not halt, meaning that it runs forever without termination. One example in which the Turing machine does not halt is producing the decimal representation of an irrational number (e.g., π), since such a procedure runs ad infinitum. The Turing machine is assumed to run without making mistakes, and it can process problems that go beyond ordinary calculation—for example, providing a yes-or-no answer for a mathematical question or a proof for a given theorem.

Recall that the computability of a problem depends on the existence of an algorithm. Since the number of existing algorithms is countably infinite, the number of computable problems and computable functions must be countably infinite. Let F be the set of all numerical functions, F_C be the set of computable functions, and F_N be the set of noncomputable ones; we have $F = F_C \cup F_N$. As we have an uncountably infinite number of all numerical functions for F, and F_C is countably infinite, F_N must be uncountably infinite. The takeaway is that the number of noncomputable problems and functions surpasses the number of computable ones, regardless of how rarely we see noncomputable problems in ordinary calculation. Turing (1937) was well aware of the existence of problems that could not be computed, as he gave examples where his depicted machine failed to terminate in a finite number of steps. However, by definition, anything that is *algorithmic* can be computed by the Turing machine, and is thus *effectively computable*. Alonzo Church (1936) developed a scheme called λ-calculus (whose technical details are beyond the scope of this article) that is equivalent to the working of modern computers, which are in turn based on the operation of the Turing machine that Turing himself imagined. The equivalence between Turing's and Church's discoveries, at least in their mathematical forms, is now referred to as the Church-Turing thesis.

Gödel's Theorems

We have shown that any problem that is algorithmic is effectively computable by a Turing machine. However, as Turing himself showed examples where his idealized machine failed to terminate, there must exist problems that are *nonalgorithmic* in nature and thus cannot be solved by machines. In 1931, Kurt Gödel (1995a), through his two famous incompleteness theorems, demonstrated decisively that mathematics is inexhaustible, meaning that there must be some domains of mathematics that are *not* algorithmic or computational. The first theorem states that there exist arithmetic sentences for which no algorithm could decide whether they are true; the second one shows that a logical mathematical system cannot prove its own consistency within that same system. A nontechnical treatment of the two theorems can be found in Nagel and Newman (1958). Hao Wang (1997, 3) expressed the two theorems combined in the following equivalent statements:

- "Any consistent formal theory of mathematics must contain undecidable propositions."
- "No formal system of mathematics can be both consistent and complete."[1]

The theorems, in other words, prove that mathematics cannot be mechanized. *Gödel overturned the belief that all mathematical and arithmetic reasoning is algorithmic, or computational*. These theorems bear several epistemological implications, and here it is necessary to quote Gödel in full:

> The human mind is incapable of formulating (or mechanizing) all its mathematical intuitions. That is, if it has succeeded in formulating some of them, this very fact yields new intuitive knowledge, for example the consistency of this formalism. This fact may be called the "incompletability" of mathematics. On the other hand, on the basis of what has been proved so far, it remains possible that there may exist (and even be empirically discoverable) a theorem-proving machine which in fact *is*

1 That is to say, a consistent system cannot decide all problems, while a system that could decide everything must be inconsistent. However, inconsistency could imply anything, true or false. Gödel's quote at the beginning of the article could be understood to say that if socialism is consistent, then it cannot solve every economic problem; if it is complete, then we cannot determine whether it works correctly or incorrectly, because in an inconsistent system, anything, true or false, could be deduced.

> equivalent to mathematical intuition, but cannot be *proved* to be so, nor even be proved to yield only *correct* theorems of finitary number theory.
>
> Either the human mind surpasses all machines (to be more precise: it can decide more number-theoretical questions than any machine), or else there exist number-theoretical questions undecidable for the human mind. (quoted in Wang 1997, 184–85)

This serves to demonstrate that only the fraction of its mathematical knowledge that the human mind is capable of formalizing can be programmed or translated into algorithms. On the other hand, the unformalized part of our mathematical knowledge consists of many mathematical propositions that we can perceive to be true but that cannot be processed by computer algorithms—including noncomputable functions, which, as we showed in the above discussion, make up a large fraction of functions.[2] To put it differently, since computers are not capable of identifying many mathematical truths that our minds can comprehend, their computational capability is decidedly inferior to that of humans. The modern version of this argument is the famous Penrose-Lucas argument, formulated in Penrose (1989, 1994, 1997), which states that there exist propositions (Gödelian sentences) whose truthfulness cannot be determined by algorithms but can be by the human mind.

Moreover, Gödel (quoted in Wang 1997, 186) elaborated on the possibility of the existence of a supercomputing machine that is equivalent to a mind: "The incompleteness results do not rule out the possibility that there is a theorem-proving computer which is in fact equivalent to mathematical intuition. But they imply that, in such a—highly unlikely for other reasons—case, either we do not know the exact specification of the computer or we do not know that it works correctly." This elaboration highlights the fact that even if humans are somehow able to design a supercomputing machine as depicted—meaning that we do know its detailed specification—*then either it will not work correctly, or, if it does, its correctness will not be comprehensible by the human mind*.

Gödel went even further, saying that the introduction of new information to a programmed procedure *complicates* its process of computation, in the sense that it adds a series of extra steps (no rigorous proof for this statement was ever made). In contrast, our minds as computing devices always aim at the *simplest* possible process with fewer steps. This is possible because the

2 The most famous unsolved problem in mathematics is probably Goldbach's conjecture that every even number greater than two is the sum of two primes. An algorithm to check the conjecture, however, would require us to check every even natural number, and such a procedure will not terminate.

human mind possesses the capacity to come up with creative approaches, *in abstract terms*, which cannot be captured by any computing machine, whether hypothetical or man-made. In the words of Gödel (quoted in Wang 1997, 189): "It would be a result of great interest to prove that the shortest decision procedure requires a long time to decide comparatively short propositions. More specifically, it may be possible to prove: For every decidable system and every decision procedure for it, there exists some proposition of length less than 200 whose shortest proof is longer than 10^{20}. Such a result would actually mean that computers cannot replace the human mind, which can give short proofs by giving a new idea."

This point exposes the crucial drawback of the idealized Turing machine: given the creative nature of the human mind, there is no deterministic end to its corresponding computing processes, since it is conceivable for the human mind to calculate problems that are *infinite* in nature. The Turing machine, on the other hand, like any programmed algorithm, must have a stopping rule. Leaving aside the ambiguity over how to determine and set such a stopping rule, an algorithm must conclude in a *finite* series of steps. This alone strengthens the claim that the computational capability of the Turing machine is and must be inferior to that of humans.

After the above discussion, we are now able to bring Gödel's insights to cast skepticism on the feasibility of central planning, whether it be market socialism or technosocialism, which significantly misconstrues the nature of complex computation as Gödel saw it. To sum up, the epistemological implications of Gödel's incompleteness theorems include the following:

- Not every mathematical problem is computational, and thus computable, and the set of noncomputable problems is significantly larger than the set of computable problems. The human mind is capable of seeing mathematical truths, including those in noncomputable problems, which is not the case for algorithmic computing machines.

- The correctness of an ideal computational machine is not provable within the system that it operates and is also not comprehensible by the human mind.

- The introduction of new information *simplifies* the computation executed by the human mind, but *extends and complicates* the computation executed by algorithmic computers.

- Computation conducted by the human mind can be extended infinitely for infinite problems, while a computing machine is necessarily finite in nature with an arbitrary stopping rule.

The Incompleteness of Market Socialism

Socialist theorists of the past such as Lange (1936, 1937) and Lerner (1934) faced different theoretical as well as practical challenges with their market socialism project. The practical challenges could be conceived of as having two parts: the transmission problem (whether it is possible to gather, aggregate, and process information from many different economic agents in a timely manner to perform economic calculation) and the computation problem (supposing that all relevant economic data could be gathered and given, whether it is possible to perform economic calculation on the massive volume of data in a highly complex economy to yield a competitive equilibrium). The computation problem was indeed recognized in 1927 by Vilfredo Pareto (2014, 117): "In the case of 100 persons and 700 commodities . . . we shall therefore have to solve a system of 70,699 equations." Hurwicz (1969, 1973) thus argued that the essential problem lay in the need for a supercomputer to carry out such a calculation.

It is clear that market socialists always conceived of their problem as a technological one, which in principle could be overcome as technology developed. Critics of market socialism responded by arguing that the problem of market socialism is never a technological one, but an economic and institutional one, since the central planner has no way to gather the relevant, contextual, and tacit knowledge of the economy that is required for the process of planning; further, the planner has no way to replicate the social appraisal process of price formation that is sine qua non to resource allocation (Rothbard 1991; Boettke and Candela 2023; Lambert and Fegley 2023).

The literature on computable economics (Velupillai 2000; Wolpert 2001), on Gödelian grounds, went a step further in arguing that there is also a computability problem in socialism that goes beyond any state of technology. The computability problem has two parts: a practical impossibility and a methodological impossibility. For the former, Koppl and Rosser (2002) and Koppl (2010), based on Wolpert (2001), showed that economic planning is impossible because planners cannot calculate and forecast the future *even if* the future is completely determined by the past. This is because economic systems face a self-reference problem, and it is an established result in logic that self-reference always leads to contradiction. This argument supports Murray Rothbard (1991), who defended Mises on the grounds that market socialism not only needs to take into account past and present prices, but also must correctly appraise future prices—an impossibility. On methodological grounds, Velupillai (2000, 2005) and Bucciarelli and Mattoscio (2021) put forth an even stronger challenge: that the impossibility of computation comes from the neoclassical roots of market socialism. Since neoclassical economic theory is built upon axiomatic choice theory, they assert, it suffers as an axiomatic system from Gödel's incompleteness theorems. As a result, the solution to the optimization problem is not just hard to compute, but could even be undeterminable. Furthermore, we cannot even show whether there exists an effective algorithm which economic agents could use to arrive at the optimum. In short, they find that social economic planning is not computable, supporting the Mises-Hayek thesis.

From the Austrian camp, Robert Murphy (2006), by using Cantor's diagonalization, also argued that it is not possible for market socialists to compute the planning problem, because the planner could not even list out all relevant prices, which he argued are uncountably infinite. That is to say, an infinite problem cannot be solved by finite algorithmic means. Allin Cottrell, Paul Cockshott, and Greg Michaelson (2007, 3–4), in response to Murphy, admitted that market socialism is the impossible type of socialism, since given the infinite nature of the constructive mathematics behind neoclassical economics, the planning problem could not be carried with finite algorithms, which are nonconstructive. However, Cottrell, Cockshott, and Michaelson (2007, 6–7) refuted Murphy's claim that prices of goods are uncountably infinite. They asserted that as we could list out all conceivable goods and their prices, they must be countable even if they are infinite. The problem is that countability does not imply computability. There is an established result in mathematical logic, Rice's theorem, which states that any computable index set (that is, where there is an algorithm to list out the enumeration of the set) must be trivial, meaning that it must be either empty or $\mathbb{N}$, the natural numbers. Ordinal preferences in economics could be conceived of as an example of a noncomputable index set. It is evident that it is not an empty set, and even if we enumerate ends on our preference scales, there is no obligation that we must enumerate based on $\mathbb{N}$. For instance, we could enumerate goods with a subset $S \subset \mathbb{N}$ due to indifference. By Rice's theorem, our preferences must be noncomputable, even if they could be countable. Velupillai (2000, 40) put it rigorously: "Given a class of choice functions that do generate preference orderings (pick out the set of maximal alternatives) for any agent, there is no effective procedure to decide whether or not any arbitrary choice function is a member of the given class."

Even though economic agents do optimize in the Misesian sense, and it is true that the function U could in principle represent our preferences, Gödel's theorems imply that such an optimization problem can only be comprehended *by our own minds* and not by a computer (Wang 1997, chap. 6) because computing devices cannot deal with noncomputable sets. In short, the root cause of the impossibility of market socialist computation is the fact that computing machines, whether they are man-made or hypothetical, by definition cannot process noncomputable functions and sets, whose mathematical truths can be comprehended by the human mind. Consequently, regardless of the volume of inputs presented to the central planner or the computing machine, it is impossible to compute economic problems for the economy.

In some ways, the above argument mirrors Mises's (1998) argument in *Human Action* (1949) that a social planner cannot calculate, because in order to do so, he needs economic data presented in terms of cardinal units. Unfortunately, there exists no device (i.e., "algorithm") to convert ordinal preferences to a common cardinal unit, leading to an impossibility. As Mises (1998, 97) convincingly put it: "It is vain to speak of any calculation of values. Calculation is possible only with cardinal numbers. The difference between the valuation of two states of affairs is entirely psychical and personal. It is not open to any projection into the external world. It can be sensed only by the individual. It cannot be communicated or imparted to any fellow man. It is an intensive magnitude."

From Market Socialism to Technosocialism

Modern technosocialist papers such as Cockshott and Cottrell (1993), Cottrell, Cockshott, and Michaelson (2007), and Dapprich and Cockshott (2023), while admitting the computational limitations of the old market socialism model, claim that those past limitations are no longer present with technosocialism. On the one hand, what differentiates the modern technosocialist proposal from the old market socialism system is that technosocialism is based not upon subjective value theory and the traditional notion of competitive equilibrium, but instead on the labor theory of value and a statistical equilibrium satisfying certain given conditions (Cottrell, Cockshott, and Michaelson 2007; Dapprich and Greenwood 2024). Specifically, the social utility function has been replaced by an objective function determined by social objectives chosen by the planning board. Here, labor vouchers substitute for consumer demand, and prices are calculated as shadow prices corresponding to technological constraints. This system will then adjust to market conditions through a feedback mechanism in which consumers reveal their preferences. Cockshott and Cottrell (1993, 165), for instance, "envisage a system in which teams of professional economists draw up alternative plans to put before a planning jury which would then choose between them." This planning procedure is executed by an input-output method fleshed out in Dapprich and Cockshott (2023), such that "economic planning can be made responsive to consumer demand through a feedback control mechanism. Output targets of products would be adjusted in response to observed consumer demand or based on predictions about future demand" (412). Furthermore, in the modern world, with modern technological advances such as data mining as well as sophisticated computational techniques like statistical learning and linear and nonlinear optimization, both the transmission problem and the computation problem are believed to be easily overcome. Data mining and data transmission allow information about economic agents to be identified, stored, and transferred in seconds, while optimization techniques can yield outputs within a short time span, even with a large volume of data.

In response, Lucas Engelhardt (2013) showcased the unrealistic time window required for socialist calculation to execute its computation process. As Engelhardt (2013) and his later adjusted model in Cwik and Engelhardt (2023, 325) pointed out, "Modern supercomputers are still not powerful enough to solve central planning's computation problem." Cottrell (2021), in his response to Engelhardt's challenge, argued first, that there are multiple computing methods (rather than just Gaussian elimination) that are time-efficient to solve the planning problem; and second, that with the exponential growth of modern technology today, the planning problem's difficulty is overstated. Although Paul Cwik and Engelhardt (2023, 343) adjusted their model to maintain their previous conclusion, they also stated that "fundamentally, the computation problem is, and has always been, a technological problem and therefore entertains technological solutions. We cannot immediately rule out the possibility that algorithms that are more computationally efficient will come along, but we also cannot immediately rule out improvements in processing speed—whether through the gradual improvements of supercomputers or through the introduction of entirely new technologies like quantum computers and the algorithms they enable." By treating the computation problem as a *technological*, and thus practical, problem, Cwik and Engelhardt need to admit the possibility of machines and algorithms outcompeting the human mind in computational capability, however unlikely it might be.

That said, while this article seconds the practical/technological challenge proposed by Engelhardt (2013), we believe that the computability problem ought to be addressed as a theoretical problem. As we shall see, from a computation-theoretic point of view, the modern technosocialist proposal based on informational and algorithmic control did not and cannot overcome the computability problem that its predecessor, market socialism, faced in the past. The inevitable conclusion is that even if all necessary and relevant economic information could be collected and put on the table for the central planner aided with a supercomputing device, and even if all of it indeed served as the correct inputs, *computation would still be impossible*.

The Incompleteness of Technosocialism

As discussed previously, market socialism inevitably faced a computability problem. Unlike market socialism, technosocialism is believed by its proponents to be capable of overcoming the impossibility of computation. In this section we argue that this is not the case, echoing Boettke and Candela (2023, 45) that technosocialism just "[put] an old wine into an irrelevant new bottle" from a computation point of view. The essence of the problem is that the Church-Turing thesis, discussed previously, is the backbone of the modern technosocialist proposal (Cottrell, Cockshott, and Michaelson 2007). Our response is twofold: first, the economy is unlikely to be a computing machine; and second, even if the economy is indeed a computing device, it is a device superior to central planning. We maintain that the epistemological implications of Gödel's incompleteness theorems serve as a sufficient refutation of the application of the Church-Turing thesis to physical systems. It is necessary to first address the implication of the Church-Turing thesis beyond its mathematical formulation.

The Church-Turing Thesis, Practical Implication

This article has shown in the first section that a problem is *effectively computable* if it could be computed by a Turing machine. However, there exists another type of computability: *intuitive computability*—that is, how people compute mathematical problems with mental processes. Turing (1937) conjectured that every function that is intuitively computable is effectively computable. To put it differently, for Turing, a human is essentially a living computer, hence the mental computational processes of a human can always be transformed into algorithms. Roger Penrose (1994, 20–21) explained that this is because Turing equated an *algorithmic* device with a *physical* device:

> It is, however, probable that Turing himself had something further in mind: that the computational capabilities of any physical device must (in idealization) be equivalent to the action of a Turing machine. Such an assertion would go well beyond what Church seems originally to have intended. . . . It seems likely that he viewed physical action in general—which would include the action of a human brain—to be always reducible to some kind of Turing-machine action. Perhaps one should call this (physical) assertion "Turing's thesis," in order to distinguish it from the original (purely mathematical) assertion of "Church's thesis."

This led to Turing's conclusion that any function that is computable by people in the intuitive sense is also computable by a hypothetical machine, the Turing machine. Turing himself also introduced the concept of a *universal Turing machine*, a machine that is capable of performing the task of *any* Turing machine. This implies that the universal Turing machine could carry out *any* algorithmic action whatsoever. Consequently, if we conceive of each individual as a single computing machine, then the solution to a computing problem given by many different individuals is indeed the same as the one given by the universal Turing machine. This is a crucial element in determining the feasibility of technosocialism, which asserts that decentralized computation in the market could be replaced by the universal computation of central planning.

One problem with the Church-Turing thesis is that it is *not provable*. A proof "would have to consider every conceivable programming language [or algorithm], [which is] not feasible. . . . Church's thesis is not a statement of mathematics, but a statement of faith that precludes the possibility of proof" (Hedman 2006, 311). Besides, as implied by Gödel's incompleteness theorems, there must exist mathematical procedures of the human mind that are not computable and thus cannot be executed by any computer, and it has been shown above that the number of noncomputable problems is significantly larger than the number of computable problems. As shown by the works of Robin Gandy, which "have analyzed idealized, discrete, deterministic machines following the laws of classical mechanics, the conclusion is that such machines cannot compute any functions that cannot be computed by humans" (Leary and Kristiansen 2015, 197).

In short, while the *mathematical* formulation of the Church-Turing thesis shares an affinity with Gödel's incompleteness result (for instance, Turing's halting problem also showed the existence of noncomputable problems), the *practical* implication of the thesis for *physical* systems goes a step beyond that. The essential divergence between the two, or what Gödel (1995b, 306) in 1972 called "a philosophical error in Turing's work," is the fact that Church-Turing equated a computing machine with the human mind, thus equating intuitive computability with effective computability.[3] It is interesting to note that Hayek (1963, 341) recognized the parallel between Gödel's incompleteness theorems and Hayek's own theory of mind, which is further developed in the self-reference problem highlighted by Roger Koppl (2010). Gödel formulated his theorems in terms of lower- and higher-order formal logic systems, and Hayek conceived the problems in terms of levels of complexity, such that "the capacity of any explaining agent must be limited to objects with a structure possessing a degree of complexity lower than its own" (Hayek 1952, 185). This similarity, however, has not been explicitly expounded.[4]

The above discussion serves as the basis for our following argument against the technosocialist proposal. Because technosocialism must rely on the Church-Turing thesis and its practical implications, drawbacks of the Church-Turing thesis inevitably raise concerns about the feasibility of technosocialism.

Is the Economy a Computing Device?

Both the market socialism of the past and the technosocialism of the present aim at replacing the "optimization" process in the market with the central computation of the planner. Technosocialists treat each individual economic agent as a computing device which solves its own optimization problem, and the central planner as a universal Turing machine which is believed to

3 The extent to which Gödel is different from Church-Turing is still debated, as some argued that the gap between Gödel and Church-Turing is not as significant as what Gödel tried to showcase (e.g., Brewer 2023, chap. 12).

4 Thus, a survey of the commonality between Hayek's theory of mind and Gödel's theory of mind is open for further investigation. For an attempt at reconciling Hayek's and Gödel's methodologies, see Van den Hauwe (2011).

yield output identical to that from many individual computations. That is, technosocialists accept the Church-Turing thesis and its physical implication as true, and then proceed with their models of computation. Unfortunately, as pointed out above, the (physical) Church-Turing thesis, which treats the human mind as a computing device, is a *statement of faith* because we cannot prove or disprove it. In other words, while the domain of Church-Turing is computational, a justification of Church-Turing requires *metacomputational*, or *hypercomputational*, consideration. Discussion about computation beyond Church-Turing, or hypercomputation, is beyond the scope of this article; readers are invited to consult da Costa and Doria (2003, 2006), for instance. For the present, we focus on the question, Can the economy be considered a computing device? We argue that there are reasons to believe that the answer is no.

Bartholo et al. (2009, 78), parallel to Hedman (2006), concluded that the answer is unknown, since "of course anything that inputs and outputs data can be looked upon as some kind of computing device, but unless we clarify its inner workings, it will be an useless computing blackbox." Even if the economy could be conceived of as a computing device, it is still the case that such a device is not equivalent to and thus could not be modeled by a Turing machine (da Costa and Doria 2006; Bartholo et al. 2009; Velupillai 2000). Such a device, which is required to have the ability to decide noncomputable problems, is a theoretical possibility, but its actual existence is still at best ambiguous (Bartholo et al. 2009, 79):

> No one has ever tried to build a device that at least approximates in some reasonable sense those conditions to see how it performs in the real world. Parts of economic systems can be modelled by linear equations, with formal solutions that only use elementary functions, polynomials, sines, cosines, exponentials. One possible line of action would be (theoretically) to add up systems modelled by adequate linear equations and to connect them in order to obtain a noncomputable predicate. . . . Would one such construction reflect something that does happen in the world of economic systems? It remains to be seen.

Gödel's theorems, on the other hand, yield the implication that not only is the human mind not equivalent to a computing device, the human mind is even superior, since no computing machine has the capacity to solve more mathematical problems than the mind. This implication could be expressed in the following propositions:

- There does not exist any man-made computing machine that exceeds the computation capability of the human mind.

- A hypothetical machine that is equivalent to the human mind, if it exists and solves problems correctly, either has its correctness unprovable within the system in which it operates, or has its correctness incomprehensible to the human mind, or both.

In the context of the market economy, this means that no (central) machine could perform the nonalgorithmic calculation of individual agents (for example, entrepreneurial judgment under uncertainty). This aligns well with the classic Austrian critique that socialism is impossible because it cannot replicate the social appraisal of prices done by entrepreneurs. Further, while both Gödel and Bartholo et al. (2009) admitted the possibility of an ideal machine that could decide noncomputable problems, Gödel suggests that the nature of the centrally computed solution would not be able to be comprehended by individual economic agents. If individuals could not epistemically comprehend the optimal nature of the solution generated by planning, it is reasonable to expect that they would not be convinced to act in accordance with it either. To put it differently, the only way to make sure that individuals act according to the centrally calculated solution is through command and control. Here the problem of socialist planning inevitably turns out to be a political problem.

Evidently, the whole concept of a democratic feedback mechanism, outlined by Jan Dapprich and Cockshott (2023), is internally an oxymoron and an impossibility. This result was also foreseen by Don Lavoie (1985, 225–26), who furthered the Mises-Hayek thesis, showing that even when the calculation problem and the knowledge problem are no longer concerns, a centrally planned economy must confront the *power problem*: "The origins of planning in practice constituted nothing more nor less than governmentally sanctioned moves by leaders of the major industries to insulate themselves from risk and the vicissitudes of market competition. It was not a failure to achieve democratic purposes; it was the ultimate fulfillment of the monopolistic purposes of certain members of the corporate elite. They had been trying for decades to find a way to use government power to protect their profits from the threat of rivals and were able to finally succeed in the war economy."

Centralized versus Decentralized Computation

As a derivative of the practical Church-Turing thesis, the belief of technosocialism in the equivalence between economic agents and computing devices, which are in turn equivalent to a universal computing device, is hard to justify. In this section, for the sake of argument, we suppose for a moment that individuals *can* be conceived of as computing devices. Then we find that not only do technosocialists believe that decentralized computation in the market can be replaced by collective planning, but Cottrell, Cockshott, and Michaelson (2007) argue that any challenge that collective computation confronts must be applied to decentralized computation as well. So if a criticism of technosocialism is correctly placed, then the market must be deficient as well. This translates the debate from a discussion regarding the possibility of central computation to one regarding the superiority of centralized computation over decentralized computation of the market. We argue that even if we see the market as a computing device, such a device is superior to the centralized computing device of technosocialism because it can handle a more complex structure due to the fact that its operation is not merely a mechanical procedure.

Lavoie, Howard Baetjer, and William Tulloh (1991), by comparing the structure of the market economy with object-oriented programming, recognized that the merit of the market qua a decentralized computing system is the fact that it can bring order to a complex setting through a process of learning and adaptation. While central planners also start with dispersed information and knowledge, the difference is that central planners need to know what they are computing. Koppl (2010, 862) fleshed this out: "Markets are not persons or goal-seeking organizations. They are the space in which such purposeful entities interact. The order they produce is defined in the process of its emergence. Because markets do not have to know what they are doing, they can reach equilibria that cannot be computed ahead of time."

In the same vein, Abigail Devereaux, Koppl, and Stuart Kauffman (2024) highlighted another advantage of a decentralized system: it is not limited to a given possibility space because it can deal with the unknowable elements of the market. This argument utilizes the role of dispersed and tacit knowledge in the economy, which, Dapprich and Dan Greenwood (2024) admitted, technosocialism cannot take into account effectively at the moment. While for collective planning computation must be made with an ex ante objective, in the market "the planning process in creatively evolving systems must cope with new parts of reality being revealed in the process of plan-realization. Since possibilities are created in the process of interacting with the system and other individuals, embedded observers are aware that they do not know everything they need to in order to plan optimally (or often satisfactorily), but they're also aware they might fill in missing steps of their plan in the process of executing it" (Devereaux, Koppl, and Kauffman 2024, 503).

Here we would argue further that decentralized computation is superior not only because it can handle a higher degree of *complexities* or the *unknowable*, but because it can deal with the *undecidable* as well. As pointed out by Penrose (1994, 153), even under a completely evolutionary algorithm, a "non-computable entity might arise out of entirely computational constituents." When this arises, the algorithm breaks down because the emergence of noncomputable phenomena does not belong to its mechanical procedure, even if it's evolutionary and adaptive. The root cause of the breakdown is the fact that *the external environment cannot supply a nonalgorithmic, noncomputable factor to the internal components*. To put it within the context of the market, not only can the central planner not forecast unintended consequences because the central model cannot adapt, *the planner cannot even understand market phenomena* if they fall into the nonalgorithmic domain. Individual economic agents, on the other hand, can understand the markets and their action through their exercise of judgment, due to the capacity of their minds to process noncomputable problems.

This suggests that technosocialists' dream of a realistic simulation of the real market is flatly unachievable. Recall that technosocialism disregards the problem of utility optimization; instead, the planning board determines an objective function based on social standards, and such a function is generated through simulation and bootstrapping (Cottrell, Cockshott, and Michaelson 2007; Dapprich and Greenwood 2024). Technosocialists talk at length about how their simulation could be computed within a short time span; however, the problem is not the *efficiency* of the simulation, but whether the simulation is *effective*. Wang (1974, 310) made it clear: "Since simulation is only on the global level, it depends on each individual's theory of how man operates in an overall manner, and does not possess the quality of faithful reproduction as suggested by the term 'simulation.' . . . In the very central area, we do not know, even in global terms, how information is selected, organized, and retrieved by the mind. Our 'pretty good idea' can be no more than crude plausible guesses."

In short, an effective simulation of the economy requires an understanding of how individual agents operate and act in the market. This requirement highlights the circularity problem that technosocialists must face: they claim that their central computing device is superior to the market, but in order to effectively design their computing device, they require an understanding of how the market and its components function, which is impossible for a central computing device because it cannot understand nonalgorithmic phenomena.

On the Role of Information

Throughout the article, we utilize the argument of Gödel that individuals can handle noncomputable problems due the creativity of the human mind. In this last subsection, some remarks about this point are warranted. Given its creative nature, the human mind is always capable of forming new ideas, whether formalizable or abstract, to solve current mathematical-computational problems in the simplest way possible with fewer steps. An algorithmic computer, on the other hand, does not possess that property. Thus, the introduction of new (and creative) information enables humans' computational processes to be more efficient. In contrast, according to Gödel, the introduction of new information *extends* and *complicates* the computational process executed by an algorithm.

One classic example of this is the case of polynomial regression. Given n data points, we can always compute a polynomial of degree $(n - 1)$ that approximates the relationship of all the data points with perfect fit. But as n grows larger and larger, it becomes much more complicated and time-consuming to compute such a polynomial. More to the point, the introduction of the $(n + 1)$th data point completely alters the previous polynomial. In a way, this procedure does not provide us any meaningful information about the relationship between the data points. Our minds, alternatively, exercise their creative power to make meaningful conjectures for mathematical objects, even though such conjectures are not perfect. The reason for this is the fact that, as Gödel demonstrated, some mathematical objects are not entirely mechanical. For example, we can conceive of real-life objects with a Euclidean-geometrical representation without being aware of their exact magnitudes. Nonetheless, meaningful mathematical truths can still be deduced from that imperfectly approximated logical system.

The implication of the above challenges modern technosocialists' claim of their capacity to compute economic data efficiently in a short time span. Even if we assume that modern technology allows the central planner to compute data from time *t immediately* at time $t + \varepsilon$ (not a reasonable assumption), the introduction of new economic information in a highly changeable, adaptive, and dynamic economic system must complicate and extend the computation process in the immediate future. A similar point was made by Jesús Huerta de Soto (2010) in his discussion about how socialism is unable to handle the dynamic change in economic knowledge and information. This point also highlights the inner contradiction of the belief of technosocialists that a supercomputer can *both* process a large amount of data in a timely manner *and* adapt to changing economic conditions.

A related problem is the possibility of the existence of an infinite problem that requires algorithms to solve in an infinite amount of steps. The human mind is, theoretically, capable of doing so due to its creativity. Nevertheless, any consistent algorithm must be concluded after a series of finite steps. The stopping rule for computation conducted by humans could be roughly thought of as arriving at mathematical truth, while any stopping rule applied to a specific algorithm needs to be arbitrarily decided and set a priori. No satisfying criteria for setting an optimal stopping rule have been proposed by the technosocialists, leaving unaddressed the question of what actually constitutes an optimal stopping rule. Besides, the process of determining the optimal stopping rule, in essence, requires exercise of *judgment* by humans, which can never mechanized or translated into an algorithm.

Conclusion

Gödel's incompleteness theorems are a beautiful and insightful result whose implications can be extended to the domain of social sciences. This article, by reexamining the two incompleteness theorems and their epistemological implications, attempts to showcase the fact that the modern advocates of technosocialism possess a significant misunderstanding of the nature of computation. In doing so, the article highlights the parallel between the incompleteness theorems and the Mises-Hayek thesis on the impossibility of socialism.

The problem identified in this article through an application of Gödel's insights—the *computability problem*—complements, and in some regards is a more extreme result than, the calculation problem identified by Mises and the knowledge problem of Hayek. This result suggests that *even if* there exists a supercomputer that is capable of generating accurate outputs, and *even if* all relevant and contextual economic information at all times and places can be gathered as correct inputs for the planning problem, socialist computation is still impossible, regardless of the state of technological development. This article also serves as a response to the critics of the Mises-Hayek thesis who argue that Mises and Hayek merely engaged in verbal discussion without any rigorous and logical reasoning to support their thesis. On the contrary, the impossibility of socialism can be shown purely from a mathematical-logical, computation-theoretic standpoint.

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