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THE AUSTRIAN SCHOOL AND MATHEMATICS: RECONSIDERING METHODS IN LIGHT OF COMPLEXITY ECONOMICS

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ABSTRACT: This article reconsiders the Austrian school's historical position regarding the role of mathematics in economics in light of complexity economics' approach to this issue. It first shows the three typical objections to the traditional use of mathematics raised by Austrian economics. Secondly, it presents complexity economics' critique of algebraic mathematics, which is employed in mainstream economics, and its proposal for using algorithmic mathematics and computation. Then, it analyzes the similarities between the Austrian and complexity economics positions and considers whether the alternative algorithmic method that complexity economics advocates is compatible with and valuable for Austrian economics. The article concludes that Austrian economics can use algorithmic mathematics to elaborate economic theory without contradicting its own methodology and that algorithmic models and simulations can in fact enlarge Austrian theories on the working and emergence of the market process.

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According to Backhouse (2000), the rejection of mathematics in elaborating theory constitutes one of the elements that distinguishes the Austrian school of economics from the mainstream. Indeed, non-Austrian economists typically regard Austrian economics as conventional theory with a puzzling and anachronistic aversion to mathematics (Vaughn 1994). Austrian economists have rejected mathematics to formalize economic theory since the very founding of the school, beginning with Carl Menger. They considered it a limited tool, inadequate to study and grasp the essence of economic phenomena in their full reality and complexity. Generally speaking, there are three reasons why the Austrians have preferred common language over mathematics: (1) mathematical language leads to a functional rather than a causal analysis, thus portraying the economy in equilibrium, from an idealistic, mechanistic viewpoint (Mayer 1994; Mises 1998); (2) mathematics is just a translation of theories previously developed in common language, so it does not add new knowledge of economic phenomena (Rothbard 1956; Boettke 1996); (3) mathematical language is no less ambiguous than common language (Rothbard 1976).

Similar to the Austrian position, what is called *complexity economics* has criticized the traditional use of mathematics in economic theory in parallel terms. W. Brian Arthur (2021b), a renowned complexity economist, argues that algebraic, equation-based formalization of economic phenomena implies a set of unrealistic assumptions that misrepresent economic reality, as if the world were in equilibrium. Note that Austrian economics is characterized by a focus on the market process rather than on equilibrium analysis, which distinguishes it from the neoclassical mainstream (Boettke 1994). Like the Austrian school, complexity economics concentrates on nonequilibrium rather than equilibrium (Arthur 2021a; Beinhocker 2007), so this explains why both Austrian and complexity economics critique the use of mathematics in economics. Indeed, Arthur (2021b) explicitly mentions the Austrian rejection of mathematics, pointing out that economists from this school aimed to create a more realistic economic theory by using common language. Still, despite its shared refusal to employ mathematics in economics, complexity economics does not automatically opt for common language as the appropriate method, as the Austrians do, but advocates different tools; for example, algorithms and computational models such as agent-based models (hereinafter ABMs) (Arthur 2020, 2021b, 2021a).

There is a vast literature on the similarities between Austrian and complexity economics (Barbieri 2013; Koppl 2009; Barkley Rosser 2015; Moreno-Casas 2021; Moreno-Casas and Bagus 2022). Both currents are presented as an alternative to the Walrasian equilibrium paradigm in economics, and both portray the economy as an emergent process integrated by heterogeneous agents with bounded rationality interacting in a decentralized way (Arthur, Durlauf, and Lane 1997; Barkley Rosser 2015). It can even be said that Austrian economics is part of the complexity movement (Potts 2000) and therefore has exerted a considerable influence on it, mainly through F. A. Hayek but also through figures such as Carl Menger, Ludwig Lachmann, and Don Lavoie, as well as through concepts such as spontaneous order (Koppl 2009; Barkley Rosser 2015). Just as the Austrian school has conditioned complexity economics, complexity economics can influence Austrian economics in some regards. These include the issue of mathematics because the schools both critique the traditional use of mathematics in economics.

This work shall investigate whether complexity economics—advocating a different kind of mathematics from what is traditionally used in economics, but still advocating mathematical techniques—can impact and perhaps modify Austrian economics' historical rejection of mathematical language. Can the assertion be made that complexity economists' proposed mathematical tools are immune to the critiques that Austrian economists have traditionally raised against mathematical formalism? If so, then the Austrians may be able to end their traditional opposition to mathematics. If not, what is the Austrian argument against this application of different mathematical tools to economics?

In answering these questions, this article hopes to promote the articulation of a mathematical version of Austrian economic theory and the creation of a new mathematical branch within the school¬, on the one hand, and debates on the compatibility of other new mathematical tools with Austrian methodology, on the other hand. For these purposes, the historical position of several important Austrian economists regarding mathematics will be presented in the next section. Then, the complexity economics approach to the issue of mathematics will be introduced, and the similarities and differences with Austrian economics will be highlighted. The final section will analyze whether the new mathematical tools suggested by complexity economists are compatible with Austrian economics and the extent to which complexity economics can alter the historical position of the Austrian school regarding mathematics.

THE AUSTRIANS ON MATHEMATICS AND ECONOMICS

As we have said above, the Austrians seem to have rejected mathematics in economics since the very origins of the school. Carl Menger, its founder, criticized the employment of mathematics in formulating economic theory. Unlike William Stanley Jevons and Léon Walras, with whom he initiated the Marginal Revolution, Menger did not use mathematics to develop his economic theory but expressed it in common language (Jaffé 1976). According to Jaffé (1976), Menger concluded that mathematics can be helpful as an expository or subsidiary tool but that genuine research should aim to study the underlying elementary causes of economic phenomena in all their complexity, and for this, he advocated an *analytic-compositive* method. The object of economic research is to discover the laws governing market phenomena and, in turn, explain those phenomena according to their ultimate determinants in humans' physiological, psychological, and social nature. Given that mathematical language cannot elaborate on these causes any further, common language is a more appropriate method, according to Menger.

A significant number of works have analyzed Menger's position concerning mathematics in economics (Alter 1986; Barkai 1996; Blanco González 2007; Mensik 2015; Reiss 2000). All of them agree that Menger's insistence on discovering the *essence* (*das Wesen*) of economic phenomena is the reason why he opposed mathematical formalism. But what did Menger mean by essence? In explaining why Austrian economists tend to criticize the use of mathematics, Jesús Huerta de Soto (1998, 84) mentions Menger's essence argument, referring to one of his letters to Walras in 1884. Right after citing Menger, Huerta de Soto points out that mathematical formalism is adequate for representing neoclassical equilibrium theory but a futile method for expressing the subjective reality of time and entrepreneurial creativity, which are dynamic processes by nature. For Austrian economics, entrepreneurship, defined as action that involves innovation, alertness, creativity, adaptation, and judgment (Foss and Klein 2012; Kirzner 1973, 1992), is the driving force of the market economy; it is the ultimate cause of economic change (Mises 1998). Accordingly, entrepreneurship triggers an open-ended process that subverts any formal optimization exercises under hypothetical conditions of equilibrium. In fact, mathematical formalism takes entrepreneurship for granted in that it assumes that economic agents possess complete knowledge and that no additional change is possible beyond equilibrium (Kirzner 1997). Yet economic forces, in essence, are not simultaneous and mechanical but sequential and dynamic due to entrepreneurship (Huerta de Soto 1998, 84).[1] Therefore, insofar as the neoclassical mathematical formalism of simultaneous equations can only explain equilibrium states and takes entrepreneurship for granted, it represents a limited tool for Austrian research goals. Mensik (2015, 9) is very explicit regarding this point:

> Menger and the Austrians tried to pursue a much more complex task as compared with what mathematical economists did. Wanting to harness the common human knowledge, which is non-mathematical, they had to do without mathematical exactness in their economics too. This seems as a deliberate choice too, one done with the intention of being more realistic rather than being artificially exact.

This is the first reason for the Austrian rejection of mathematics mentioned in the introduction. Recall that it was pointed out that mathematics leads to a functional rather than a causal analysis, thus portraying the economy in equilibrium, from an idealistic, mechanistic viewpoint. This assertion assumes a distinction between two approaches to economic issues: a *functional* perspective and

[1] Huerta de Soto (1998) uses the term *sequential* as a contrast to continuity, referring to the praxeological idea that action takes place in time and is discrete, so that goods are not all valued at the same time but one after another. This is one of the Austrian critiques of the neoclassical assumptions of rationality and consistency, which state that if A is preferred to B, and B to C, then A is preferred to C, and any deviation from this choice is irrational. Against this reasoning, the Austrians argue that the preferences for A over B and B over C manifest at two different points in time, so it is perfectly possible and rational for an individual to change his preferences before comparing A and C.

a *causal-genetic* approach (Mayer 1994). The functional approach presupposes equilibrium conditions and thus describes relations between variables simultaneously determined. In contrast, the causal-genetic approach rejects the notion of simultaneous determination and attempts to explain economic phenomena (and even equilibrium states) by looking at their emergence and development from a causal perspective.

Austrian economists fall under the causal-genetic approach, as they emphasize the *market process* rather than economic equilibrium. From this causal-genetic stance, they criticize the functional approach and identify it with the mathematical method (Mayer 1994; Cowan and Rizzo 1996). They argue that mathematical formalism, with its simultaneous equations, sees the economy in equilibrium, in stasis. Because all magnitudes are assumed to determine each other simultaneously, the economist is unable to determine the underlying causal processes. The economy is thus presented unrealistically, as if it were in a state of rest. Moreover, since the mathematical treatment of economics is based on a mechanical analogy borrowed from classical physics (Mirowski 1989), it fails to analyze purposeful human action. Mathematical economists assume constant relationships between economic elements, just as natural scientists assume them between physical or chemical elements. However, there is no such thing as constant quantitative relationships in the field of human action. Human behavior is variable and depends on individuals' preferences (Mises 1977, 1998). Assuming constant relationships, as mathematical economists do, implies an incorrect conception and representation of reality.

For the Austrians, the principal deficiency of mathematical economics is the omission of the market process. The tools mathematical economists use, such as differential calculus and simultaneous equations, can only represent an equilibrium state that excludes change, time, and human action itself. Even the introduction of a time variable into equations does not solve this problem, since the rest of the parameters remain constant. Mathematical economics focuses on the optimization of resources that are already given, that are already in equilibrium. Consequently, mathematics cannot explain the processes moving the economy into an endless disequilibrium state based on the actions and preferences of the individuals who participate in the market. This is precisely because mathematics describes the equilibrium state that is reached once the market process stops (Mises 1998; Moorhouse 1993).

In contrast to mathematical language, common language makes it possible to discuss how entrepreneurs, promoters, speculators, or capitalists, seeking to profit from discrepancies between prices in the market, tend to eliminate those discrepancies and thus the source of business profits and losses (Mises 1998, 353). This is precisely what Austrian economists see as the task of economic theory: to explain economic processes, not equilibrium. Hence, the mathematical method can be seen as a static method, while the use of common language corresponds to a dynamic method (Mises 1998).

Austrian economists' second critique of mathematics in economics is that mathematics contributes nothing to explaining and describing economic phenomena. This is because all knowledge included in an equation necessarily has a nonmathematical character: formulating an equation only serves to represent knowledge already possessed and discovered through common language (Mises 1998). The mathematical representation of economic ideas is a mere translation of knowledge. Rothbard (1956, 1976, 2009) even warns that this translation from common to mathematical language, which is implicit in any mathematical articulation, violates the fundamental scientific principle of Occam's razor. When an economic theory written in common language is translated into mathematical symbols, the theory's components are unnecessarily multiplied, complicating any scientific explanation.

Lastly, the third critique is that common language can be as precise as mathematical presentation in expressing verbal logic (Menger 2003; Rothbard 1976). What is more, the axiomatic-deductive method of praxeology, relying on verbal logic, would lose meaning at each step of the deductive process if it were represented by mathematical language (Rothbard 1976). In physics, for instance, axioms and their deductions are purely formal and only acquire meaning operationally insofar as they serve to explain and predict given events. In contrast, in praxeology, axioms are true and meaningful in themselves because they are self-evident. Mathematical axioms lack meaning by themselves; their predictive capacity determines whether they are operationally meaningful. This is an advantage of verbal logic as compared to mathematical logic (Rothbard 1976).

In the same vein, contemporary Austrian economist Peter Boettke has employed the syntactic-semantic distinction to make clear this Austrian critique of mathematics in economics (Boettke 1996, 1997). He argues that mathematics guarantees *syntactic* clarity but not *semantic* clarity. This means that although mathematical language is understood by most scientists—which is why mathematical economists advocate mathematics to avoid ambiguity—the meaning of economic theories can be lost when expressed in mathematical language. Consequently, economic theories can easily be exchanged through mathematics but become stripped of true economic meaning in the process.

To summarize, Austrian economists typically use three arguments to reject mathematics in economics. The main Austrian critique of mathematics is directed toward the (1) static representation, in equilibrium, of economic phenomena, ignoring the market process, which requires analysis from a dynamic and causal-genetic viewpoint. Additionally, two significant questions come to the fore in discussion about the suitability of the mathematical method for economics: namely, whether mathematics allows the discovery of new knowledge of economic phenomena or is merely a method of translating common-language logic and whether mathematics is a more precise tool than common language. The Austrians conclude that (2) mathematics cannot add new knowledge of economic phenomena because it is a mere translation and that (3) mathematics is no less ambiguous than common language.

Those three points explain why Austrian economists have produced most of their theories and models without the aid of mathematical language. However, and perhaps as a surprise for many, there are exceptions to this rule. One can find a mathematical treatment of some economic explanations in Hayek (1931), Lachman (1978), and Garrison (2001), to name a few. Even the most ardent Austrian critic of mathematics in economics, Murray Rothbard, used mathematical formulation several times in his treatise to explain marginal productivity laws (Rothbard 2009). In fact, recent literature analyzing the issue of mathematics and Austrian economics has concluded that Austrian economists actually did not reject mathematics tout court (Linsbichler 2021); some authors have even advocated the mathematization of Austrian theory (Hudík 2015), while others have already mathematized some Austrian concepts (Albrecht 2016; Hudik 2020; Littlechild 1979; Littlechild and Owen 1980; Yates 2000). As will be shown in the final section, contrary to what is commonly held, Austrian economics is open to a type of mathematics that is able to capture the market process, the *essence* of economic phenomena, to put it in Mengerian terms (Mayer 1994; Huerta de Soto 1998). This is precisely the type of mathematics advocated by complexity economics, which shares the same arguments against conventional mathematical economics as Austrian economics but also proposes mathematical tools such as algorithms and computational models to explain economic processes beyond equilibrium states.

COMPLEXITY ECONOMICS AND MATHEMATICS

The term *complexity economics* was coined by economist W. Brian Arthur to designate a new approach in economic science based on the so-called complexity theory developed in physical science in the second half of the twentieth century (Arthur 1999). Before the appearance of complexity theory, a mechanistic vision of the world predominated both in science and in economics. Newtonian mechanics conceived the world as a vast clockwork mechanism. In that world, every physical phenomenon can be predicted with certainty. Moreover, given that either there is no motion or motion is reversible in time, a state of equilibrium appears when all forces in the system sum to zero (Wible 2000). Since the discipline originated as a metaphorical adaptation of mid-nineteenth-century physics to economics (Mirowski 1989, 1991), neoclassical economics adopted this mechanistic vision of the world, thus studying the economy from a static, mechanistic, equilibrium view.

Complexity theory posits that the world is much more complex than a clockwork mechanism and consists of emerging, dependent, and changing patterns. Because of their complexity, nonmechanical and complex dynamic systems can be in states of nonequilibrium (Wible 2000). Complexity economics conceives the economy as "process dependent, organic, and always evolving" and not as "deterministic, predictable, and mechanistic" (Arthur 1999, 109). Hence, it rejects assumptions such as decreasing returns, perfect rationality, and static equilibrium while emphasizing increasing returns, bounded rationality, learning, and evolution (Wible 2000).

Complexity economics criticizes the "equilibrium" and "dynamical systems" perspectives underlying neoclassical economics for not considering the emergence of new types of state variables, new entities, structures, or patterns (Arthur, Durlauf, and Lane 1997). In short, "complexity economics sees the economy not as mechanistic, static, timeless and perfect but as organic, always creating itself, alive and full of messy vitality" (Arthur 2021a, 143).

Complexity economics is not a school or current, but many regard it as a broad movement formed by diverse heterodox schools of economic thought: behavioral, experimental, evolutionary, institutional, Marxian, post-Keynesian, and Austrian economics (Potts 2000). As such, there is no universal definition of complexity in science or economics. Yet, the essence of the complexity idea can be understood with Herbert Simon's definition of a complex system: "one made up of a large number of parts that interact in a nonsimple way. In such systems, the whole is more than the sum of the parts" (Simon 1991, 468).

In addition to rejecting the mechanistic neoclassical vision of the economy, complexity economists have upheld alternatives to the tools that orthodox economists traditionally used, which represented the world statically. Thus, complexity economists' economic modeling was based on computation and algorithms rather than being an algebraic, equation-based representation of economic theory (Arthur 2021a, 2021b; Velupillai 1996, 2000, 2005; Zambelli 2012). Arthur (2021b) explains this in depth in his recent work.

Arthur (2021b) begins by stressing the many valuable methods of understanding in economics: narrative discourse, geometry, computation, economic history, and statistics. To him, all these tools investigate and show different things, and thus are all legitimate. For these reasons, he clarifies that critiquing equation-based economics does not mean rejecting but limiting its application to a specific analytical sphere (equilibrium analysis). Arthur argues that algebraic mathematics deals only with quantifiable things and does not deal with actions and processes. Hence, its use in formal economics confines the discipline to dealing with prices, consumption, quantities produced, and rates of inflation. However, the actions of agents are also central to economics. As Arthur (2021b, 2) puts it, "Investors, producers, banks, and consumers act and interact incessantly. They trade, explore, forecast, buy, sell, ponder, adapt, invent, bring new products into being, start companies. And these of course are actions." There are actions behind the quantities that economics formalizes in equations. Even so, formal economics has omitted the explanation of these actions by using algebraic mathematics, which Arthur contends can only express relationships among quantifiable parameters.

According to Arthur, the consequences of the restriction to quantities that algebraic mathematics causes in economics are that, (1) anything related to formation or process is left out; (2) actions are hidden within unspecified quantitative linkages; (3) quantities are idealized in frozen concepts; (4) equilibrium is assumed; (5) the universe becomes closed and novel creations can barely emerge; and (6) agents are endowed with perfect rationality. These are precisely the assumptions that the complexity perspective tries to overcome with its emphasis on bounded rationality, novelty, learning, out-of-equilibrium dynamics, process, and emergence (Arthur 2021a; Holt, Barkley Rosser, and Colander 2011). Therefore, Arthur, convinced that mathematics is a valuable and powerful instrument for understanding in economics, proposes an alternative tool: algorithms and computation. He argues that algorithms and computation allow the study of actions; they account for the process, not just a state of equilibrium. As sets of instructions, algorithms can express an action or process in each step, or operation. They allow for *if-then* conditions, which can contain, trigger, inhibit, or create other processes. They allow to express relationships among quantities in addition to actions. Moreover, it is worth mentioning that algorithms exist in themselves and can represent economic theory without a physical computer. Even so, running them on a computer enables economists to reach conclusions that would be unreachable with the sole use of analytic tools or through thought experiments.

Arthur points out that there are two mathematical orientations in economics: (1) algebraic mathematics, dealing with allocation and optimization problems, and (2) algorithmics and computation, dealing with formation and actions. Although they both complement each other, he does not believe the algorithmic expression is a panacea in economics. He indeed identifies some reservations and limitations of this computational way of economic thinking. For instance, it is crucial to bear in mind that algorithms and computational models are simulations, so they depend on the assumptions introduced by the modeler. They may lead to erroneous conclusions if not built with honesty and rigor. Additionally, neither algorithms nor algebraic equations can "easily capture the *humanness* of economic life, its emotionality, its intuitive nature, its personages, its very style" (Arthur 2021b, 10). For this purpose, economics would need other tools, such as common language.

To better understand the differences between the algebraic mathematics typically used in mainstream economics and the "algorithmic mathematics" proposed by complexity economics, consider the two most common types of mathematical models employed in each case.

In mainstream economics, dynamic stochastic general equilibrium (DSGE) models are the standard forecasting and policy analysis tool. These models apply general equilibrium theory and some neoclassical microeconomic principles to understand economic growth, business cycles, policy effects, and technological and market shocks, among other phenomena (Hoefman 2020). These models are based on mainstream neoclassical assumptions such as rational expectations, symmetric information, and that firms are identical price takers (Annicchiarico 2010). DSGE models start by defining some algebraic relationships between microeconomic magnitudes: household consumption functions, the labor supply equation, the production function, the capital accumulation equation, government budget constraints, tech spending, a stochastic process, etc. Then, equations are optimized by making them subject to a series of restrictions (i.e., budget constraints), and thus, all equations and parameters combined output several possible equilibrium states. In this way, DSGE models imply a linearization of variables and assume a simple correspondence between micro and macro levels, which means that equilibrium is a mere aggregative result of the combination of microeconomic interactions (Fagiolo and Roventini 2017).

The algorithmic mathematics of complexity economics is best exemplified by the so-called agent-based models (ABMs). ABMs appeared in economics as an alternative to standard tools such as DGSE models (Fagiolo and Roventini 2017). The latter were criticized for assuming a perfect world (Farmer and Foley 2009), while the former were preferred for presenting a less constrained mathematical way to explore economic issues that accounts for process and out-of-equilibrium dynamics. ABMs begin with a population of agents given explicit rules of behavior that interact directly with one another, leading to emergent macrobehavior and patterns (Axtell and Farmer, forthcoming). In contrast to traditional systems of mathematical functions employed in economics, ABMs do not prespecify any aggregate level, macrobehavior, or equilibrium. Macrobehavior emerges from agent interaction, previously characterized or programmed by the modeler (Hoefman 2020). Moreover, microbehavior is not defined as a linear, algebraic function but is encoded in algorithmic forms in computer programs, according to a set of *if-then* conditions. Network representation is also fairly common in these models. Because of their algorithmic form, then, ABMs are able to capture complex phenomena that traditional DSGE models cannot perceive due to their algebraic structure.

Placed side by side, it is not difficult to see how similar the Austrian and complexity critiques of the orthodox use of mathematics in economics are.[2] Both recognize that explanations of economic phenomena cannot be limited to equilibrium analysis, so they criticize the traditional mathematical treatment of economic theory, which inevitably leads to an equilibrium framework with linear functions. However, unlike the Austrians, complexity economists propose alternative mathematical techniques based on algorithms and computation for economic theory. This leads back to the central questions of this work: Can it be asserted that complexity economists' proposals for new algorithmic mathematical tools are compatible with the traditional Austrian position?

COMPLEXITY ECONOMICS, AUSTRIAN ECONOMICS, AND MATHEMATICS: THE END OF A HISTORICAL HOSTILITY?

Recalling the three arguments against mathematics posed by Austrian economists, it can be established that in order to be

[2] This is unsurprising, since Austrian economics has many points in common with complexity economics, especially through F. A. Hayek. There is a vast literature on the similarities between Austrian economics and complexity theory in general (Lavoie 1989; Koppl 2000; 2006; Montgomery 2000; Barkley Rosser 2010, 2012, 2015; Barbieri 2013; Moreno-Casas 2021) and much scholarship focusing on Hayek's work in particular (Chaumont-Chancelier 1999; Fiori 2009; Gaus 2007; Kilpatrick 2001; Lewis 2012; Vaughn 1999; Vriend 2002; Weimer 1982).

compatible with Austrian economics, any mathematical method must meet the following requirements: it must (1) be able to capture the dynamism of the market process; (2) discover additional knowledge beyond being a mere translation; and (3) provide at least the same semantic clarity as common language. Then, to analyze whether complexity economics' proposal can change or influence the Austrian position, it should first be checked whether the kind of mathematics proposed by complexity economists, called "algorithmic mathematics," meets these requirements.

Regarding the first argument (1), it is essential to note that Austrian economists hold an identical position to that of complexity economists. Earlier it was noted that Arthur (2021b) pointed out that his critiques of algebraic mathematics do not imply a complete rejection of this method but rather its confinement to equilibrium analysis. Algebraic mathematics is helpful in optimization and equilibrium theory, but the error of mainstream economics is to believe that all economic realities can be explained through this method. Accordingly, complexity economics presents algorithmic mathematics as a complement to algebraic mathematics: as a method to explain nonequilibrium phenomena—actions and processes.

Neither do many Austrians believe mathematics is inadequate for economics per se, as mentioned earlier. Hans Mayer (1994), for example, fiercely criticized the functional approach but did not reject mathematics as such. He opposed a vision of the economy that exclusively uses mathematics. However, he clarified that a correct view of the economy could still use mathematics insofar as a *causal-genetic* description of economic phenomena is provided. Ludwig von Mises (1998) conceded that it is possible to represent his *evenly rotating economy* by means of differential equations and curves. Huerta de Soto (1998) stated: "Mathematical formalism is especially adequate for expressing the states of equilibrium that the neoclassical economists study" (Huerta de Soto 1998, 84). He even adds:

> In any case, the mathematicians' response (if they can provide one) to the challenge of conceiving and developing a whole new "mathematics" able to include and allow the analysis of the human being's creative capacity with all its implication, without resorting, therefore, to the assumptions of constancy that come from the world of physics and which have been the driving force behind all the mathematical languages known to date, is still pending. (Huerta de Soto 1998, 100)

A complexity economist can now say that the complexity approach is the "mathematicians' response," the new mathematics to which Huerta de Soto refers being precisely what complexity economics employs.

As explained in the previous section, algorithmic mathematics aims precisely to go beyond equilibrium and include actions, dynamism, process, and emergence in economic analysis. This is clearly seen in the case of ABMs: their output is *emergent* in that the model is self-organized; interaction is dispersed; there is bounded rationality in heterogeneous agents; and there is no global controller. All these features of the model agree with Austrian economic methodology. What is more, the compatibility of these kinds of algorithmic mathematical models with Austrian economics methodology has already been studied (Vriend 2002; Nell 2010; Seagren 2011; Koppl 2006; Lavoie, Baetjer, and Tulloh 1990), with some of these authors concluding that Austrian economics should adopt agent-based computational modeling to complement and expand economic theorizing. As Seagren (2011) points out, the typical features of ABMs (described in Epstein 2006) overlap with the three core methodological tenets of Austrian economics: methodological individualism, subjectivism, and the notion of the market process (Boettke 1994).

Seagren (2011) argues that agent-based simulations can complement Hayek's *spontaneous order* economics and can even parallel Menger's *compositive* method insofar as ABMs capture the process of causation of macro-level phenomena starting from the actions of individuals. ABM simulations may facilitate understanding of the operation of complex phenomena and the process through which different outcomes may emerge. Additionally, ABMs allow for more realistic assumptions in economic models and theories. Indeed, this has been explicitly recognized by Austrian economists such as Don Lavoie, Howard Baetjer, and William Tulloh (1990) and Guinevere Liberty Nell (2010). Seagren (2011) highlights that ABMs can help Austrian researchers elaborate *conjectural history* by examining complex processes more easily.[3]

[3] In Austrian methodology, conjectural history is contingent theory, which cannot be claimed to be universal and necessary. This kind of theory contrasts with praxeological laws, which are indeed universal and necessary (Selgin 1990).

The concrete case for ABMs, which are algorithmic mathematics par excellence, shows that a new type of mathematics can capture the dynamism of the market process. As Seagren (2011) points out, ABMs can assist in the elaboration of *imaginary constructions* and *thought experiments*, as these models make it possible to control and process more variables than the human mind can manage, relaxing some ceteris paribus clauses traditionally used for the sake of analytical (algebraic) treatment—and this assertion is true of all types of algorithmic mathematics. This point refers directly to Mises (1998), who explicitly affirms that the method of economics is the method of imaginary constructions. As mentioned above, Mises criticizes mathematical economics for not accounting for the market process and for concentrating only on equilibrium. Algorithmic mathematical models can assist precisely in explaining emergent phenomena of the market process itself.

So far, it has been shown that the algorithmic mathematics proposed by complexity economics is immune to the Austrian critique that math only describes equilibrium states and even constitutes an adequate alternative to orthodox economic methods based on Austrians' description of the ideal mathematics for economic analysis (Mayer 1994; Huerta de Soto 1998), as well as Austrian economics methodology (Seagren 2011).

The fact that algorithmic mathematics enables economists to simulate and understand particular complex dynamics, which would otherwise be unimaginable for the human mind due to its limited cognitive capacity, means that this mathematical technique indeed expands knowledge of economic phenomena. Simulations are not mere translations of theories previously thought out by rough deduction and in common language, as algebraic functions are. As mentioned above, in simulations such as ABMs, the solution is not already contained in the initial equations or instructions. On the contrary, outcomes emerge and cannot be predicted or anticipated due to the complex phenomena occurring once the model is running (Axtell and Farmer forthcoming). The second Austrian argument points out that using algebraic mathematics does increase economic knowledge. Yet, as shown, algorithmic mathematics and its simulations can uncover new knowledge which may be unattainable with the sole use of thought experiments. Thus, complexity economics' algorithmic mathematics is immune to another typical Austrian objection. Algorithmic mathematics does not violate the scientific principle of Occam's razor; more than mere translation, it is a legitimate mathematical technique that Austrians can use to discover economic knowledge.

To be sure, the fact that algorithmic mathematics and its methods enlarge economic knowledge beyond the possibilities of mere analytical tools or thought experiments does not mean that this type of mathematics transcends the limits of knowledge. This is a relevant point for Austrian theory because it emphasizes tacit, dispersed knowledge (Hayek 1937, 1945) and radical uncertainty (Lachmann 1986). There are unavoidable limits to what humans can know about the world, and complexity economics' tools are equally subject to these limits. In fact, Austrian economist Roger Koppl (2010) has shown, based on Hayek's (1952) connectionist theory of mind, that one implication of economic complexity is the lack of algorithmic access to the rules governing consciousness. Because there are indeed epistemic limits to what can be modeled in physical language, it is important to clarify that when the tools of complexity economics are characterized as allowing economists to reach conclusions beyond the possibilities of mere thought experiments, this means that these tools can help control and process more variables than the human mind can manage, relaxing some ceteris paribus clauses. But it does not mean that complexity economics can overcome natural epistemic limitations.

Finally, there is the third Austrian argument: that of ambiguity and syntactic or semantic clarity. The Austrians hold that mathematical language and common language can be used with the same clarity, so there is no advantage in using mathematics. The example of ABMs can also be used to clarify this matter.

ABMs do not usually contain equations. That is why they are contrasted with equation-based, algebraic models. Granted, this does not mean that there are never equations in ABMs. In fact, as Epstein (2006) stresses, any ABM can be cast as an explicit set of recursive functions. The problem is that these formulas might be extremely complex to interpret and cannot be solved analytically but only approximately, using computer simulations. Still, simulations are best used as an inquiry tool for discovering emerging patterns rather than serving a communicative function. Modelers may introduce equations to explain some part of the simulated process, but models and their results are still described verbally or through pictorial representations (Epstein and Axtell 1996), which allow researchers to convey the proper interpretation of their models. In this sense, complexity economics does not argue that algorithmic mathematics should serve as a scientific communication tool, contrary to what mainstream economics thinks of algebraic mathematics. Like the Austrians, complexity economists rely on common language to describe and present their models so that they do not lose semantic clarity. Hence, the complexity proposal of algorithmic mathematics is compatible with Austrian economics' requirement of avoiding ambiguity in expressing economic theory.

The example of ABMs shows that the algorithmic mathematics advocated by complexity economics is compatible with Austrian economics insofar as all the typical Austrian critiques of the mathematical method do not apply to complexity's techniques. And to illustrate this compatibility, a concrete example of an ABM—an algorithmic mathematical model—elaborated by Austrian economist Per Bylund (2015) can be presented.

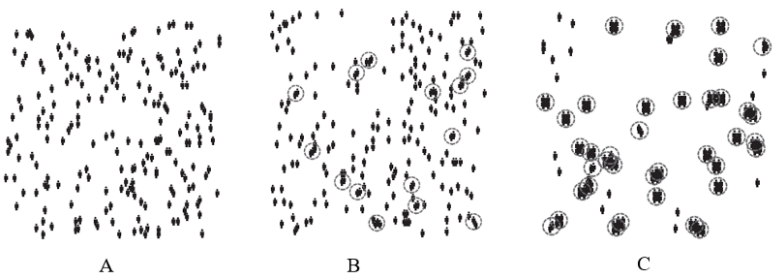
Bylund (2015) draws on an ABM to illustrate a theoretical question that does not pertain to the Austrian economics as such: the relevance of Oliver Williamson's contribution to Ronald Coase's transaction cost theory. Bylund's paper aims to test a series of propositions related to Coase's transaction cost approach to see whether Coasean transaction costs are sufficient to explain firm formation. Hence, he designs an ABM that simulates a marketplace containing independent agents who seek to trade for profit, which is a scheme akin to Coase's atomistic competition. There are two hundred agents, and they are distributed randomly on a 100 × 100 (2-D) torus space.[4] All agents are randomly assigned a specialized task in a production process consisting of five production stages: (0–1), (1–2), (2–3), (3–4), and (4–5). The production process is thought to start at 0 and end at 5, so value is added as the production process progresses. In this production process, agents buy the output of the preceding stage and sell inputs to the following

[4] As Bylund (2015) explains, a torus space is a space that is never ending but has a limited number of positions. In Bylund's model, the torus space is 100 × 100 (10,000). A height and a width of one hundred means that there are one hundred positions across the space from top to bottom and left to right. In this case, vertical position 101 at the bottom is also the first position at the top. The same logic applies horizontally, position 101 on the right being the first position on the left.

stage, behaving as profit maximizers and moving around the model space to find possible trading partners. Initial funds, inputs, and locations are randomly determined before running the model. Agents are also randomly endowed with a different risk aversion.

The purpose of the model is to see whether agents tend to create firms (defined as a stable relationship between at least two agents) under low and high transaction costs, which are signified by the cost of moving along the model space. This was tested by running over one hundred simulations of the model with low– and high–transaction cost environments. Figure 1 illustrates three different runs of the model. Case A is the basic model structure, the starting point before running simulations, including all the initial conditions mentioned above. In case B, the *entrepreneur* (or innovator) and the employee figure have been introduced, so case B reflects how firms begin to form (represented as circles containing multiple agents) as the simulation progresses. Finally, case C represents a late stage of the simulation in which the firm creation process intensifies.

Figure I. Evolution of Bylund's (2015) agent-based model over three simulations



Source: Bylund (2015).

Bylund's main finding is that specialization cannot be dismissed as an important factor for firm creation, contrary to traditional Coase's theory. The ABM allowed different transaction cost scenarios to be classified and made it possible to study their emergent dynamics—the extent to which firms were created. These types of analyses are beyond human observation or the possibilities of thought experiments. They could not be carried out without ABMs.

According to Bylund (2015, 154), the reason for using an ABM is to "effectively test dynamic causes and mechanisms that bring about phenomena that we observe empirically, but cannot fully identify or comprehend only by observation." This statement can be directly related to the major Austrian critiques of mathematics. Bylund points out that ABMs are a dynamic method (critique 1), enabling spontaneous-order-type explanations. In addition, he notes that ABMs explain phenomena that "we observe empirically, but cannot fully identify or comprehend only by observation" (Bylund 2015, 154), so they indeed enlarge our knowledge of economic phenomena (critique 2). Furthermore, Bylund does not use any algebraic functions to design the model or present results but rather utilizes common language and some pictorial representations to do it, which guarantees semantic clarity (critique 3). This concrete ABM is therefore an algorithmic model that overcomes the three most common Austrian critiques of mathematics and thus constitutes an appropriate tool for the Austrian school to use to advance economic research.

The most important implication of what complexity economist W. Brian Arthur calls "algorithmic mathematics"—a kind of mathematics that is immune to the usual Austrian objections to mathematics—is the possible end of Austrian economics' historical hostility toward the mathematical method. It is essential to emphasize again that many renowned Austrians did not reject mathematics entirely, but only the algebraic mathematics used in neoclassical economics. In this sense, Austrians left the door open to a new mathematical method compatible with their approach even as they eschewed algebraic mathematical formalization and continued to use common language. Now, with the appearance of the algorithmic mathematics advocated by complexity economics, it seems that the Austrians have no excuse not to employ mathematical tools in economic theory. It thus appears possible to be nearing the end of the historical hostility that has set the Austrian school apart from the rest of the economics profession from the beginning.

CONCLUSION

After showing that the so-called algorithmic mathematics proposed by complexity economics is compatible with Austrian economics' three common requirements and that it can be a potent tool of inquiry for this school of thought, it is possible to envisage the end of the historical hostility between Austrian economics and mathematics. The Austrian school's very founder preferred common language over mathematical language in economics, and this became the position of most of Menger's followers. However, thanks to the development of complexity economics, which was influenced by Austrian forerunners such as F. A. Hayek (Koppl 2009), some of the mathematical tools employed in economics became more algorithmic and less equation based, paving the way for a generation of models (e.g., ABMs) that can capture the dynamism of economic reality, which is the goal of Austrian economics. This article has shown how Austrian economists can use complexity economics' algorithmic models, such as agent-based models, their theory without falling into any methodological contradictions. Austrian economists can make use of computation and algorithmic models to elaborate economic theory.

This conclusion is historically relevant because it prompts the Austrian school to change its long-held position regarding mathematics and thus finally clears the way for the use of specific helpful mathematical techniques that can help expand understanding of the market process. Although most Austrians continue to reject algebraic mathematics, they seem to have no reason to oppose the use of algorithmic mathematics in economics. Now that it has been established that algorithmic mathematics is compatible with Austrian economics, it is necessary to test the limits of algorithmic models according to Austrian economics' methodology. Austrian economists should start using these tools to test and strengthen the Austrian theoretical framework and uncover knowledge that has been beyond the grasp of their original methods. The introduction of algorithmic mathematics into Austrian economics will considerably improve the theoretical richness of the school and, possibly, help it to gain recognition within the economics discipline itself.

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